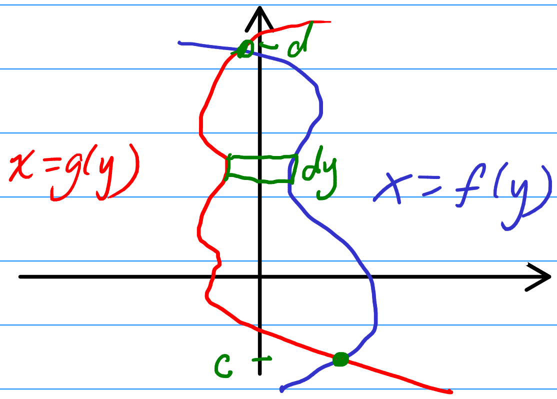
Lesson 13 – Area between curves, part II
I Area between $x=f(y)$ and $x=g(y)$

Monday – Notes Template for Lesson 14.

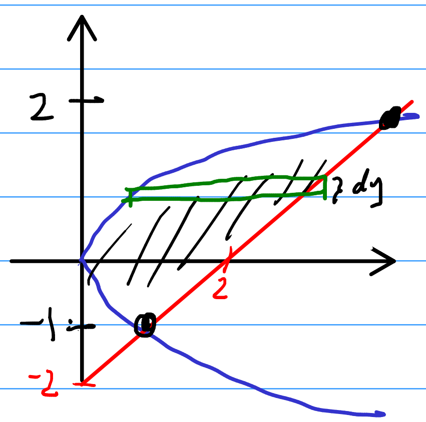
I Area between $x=f(y)$ and $x=g(y)$



$$A=\int_c^d (f(y)-g(y))\,dy$$

big – small
right – left

$\boxed{\text{EX}}$ Find the area between the curves
① $x=y^2$ and $x=y+2$



intersection pts

$y^2=y+2$

$y^2-y-2=0$

$(y-2)(y+1)=0$

$y=2, \quad y=-1$

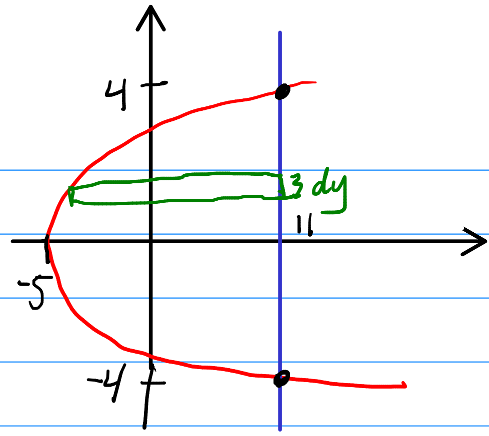
$x=4 \qquad 1$

$$A=\int_{-1}^{2}((y+2)-y^2)\,dy=\int_{-1}^{2}(y+2-y^2)\,dy$$

$$=\frac{y^2}{2}+2y-\frac{y^3}{3}\Big|_{-1}^{2}=\left(2+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2-\frac{(-1)}{3}\right)$$

$$=8-\frac{9}{3}-\frac{1}{2}=5-\frac{1}{2}=\frac{9}{2}$$

② $x = y^2 - 5$ and $x = 11$



Intersection Pts

$y^2 - 5 = 11$

$y^2 = 16$

$y = \pm\sqrt{16}$

$y = \pm 4$

$$A = \int_{-4}^{4} \left(11 - (y^2 - 5)\right) dy = \int_{-4}^{4} (16 - y^2)\, dy$$

$$= 16y - \frac{y^3}{3} \Big|_{-4}^{4} = \left(16(4) - \frac{4^3}{3}\right) - \left(16(-4) - \frac{(-4)^3}{3}\right)$$

$$= \left(64 - \frac{64}{3}\right) - \left(-64 - \frac{-64}{3}\right)$$

$$= 64 - \frac{64}{3} + 64 - \frac{64}{3}$$

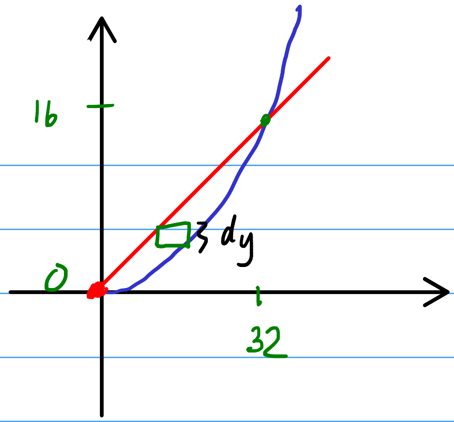
$$= 128 - \frac{128}{3}$$

$$= \frac{256}{3}$$

③ $x = 8\sqrt{y}$ and $x = 2y$

$x^2 = 64y$

$y = \frac{x^2}{64}$



intersection pts

$(8\sqrt{y})^2 = (2y)^2$

$8\sqrt{y} = 2y$

$64y = 4y^2$

$0 = 4y^2 - 64y$

$0 = 4y(y-16)$

$y = 0 \qquad y = 16$

$$A = \int_0^{16} (8\underbrace{\sqrt{y}}_{y^{1/2}} - 2y)\, dy$$

$$= 8 \cdot \frac{2}{3} y^{3/2} - \cancel{2}\frac{y^2}{\cancel{2}} \Big|_0^{16}$$

$$= \left(\frac{16}{3}(16)^{3/2} - 16^2\right) - (0 - 0)$$

$$= \frac{16(64)}{3} - 256$$

$$= \frac{1024}{3} - \frac{(256)(3)}{3}$$

$$= \frac{1024}{3} - \frac{768}{3}$$

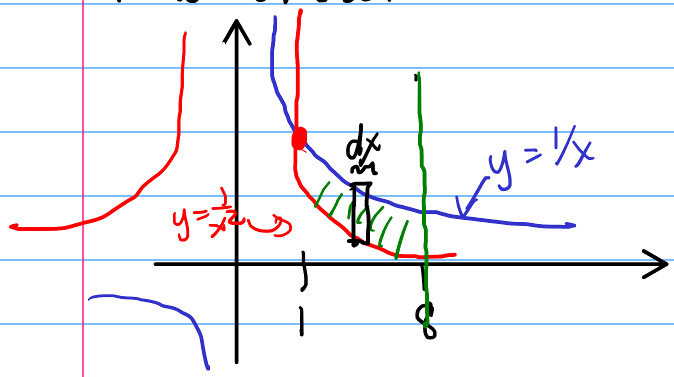
$$= \frac{256}{3}$$

Homework Questions?

HW 12 #6

$y = \frac{1}{x}$ $\qquad y = \frac{1}{x^2}$ $\qquad x = 8$

Find area.



$$\frac{1}{x} = \frac{1}{x^2}$$
$$x = x^2$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$\cancel{x = 0} \qquad x = 1$$

because x was denominator

$$A = \int_1^8 \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$$

HW 11 #4

$$\int_1^{\infty} \frac{8e^{-4\sqrt{x}}}{7\sqrt{x}} dx$$

$$\int \frac{8e^{-4\sqrt{x}}}{7\sqrt{x}} dx = \int \frac{8e^u}{7} \cdot -\frac{1}{2} du = -\frac{4}{7} \int e^u du$$

$$= -\frac{4}{7} e^u + C$$

$$= -\frac{4}{7} e^{-4\sqrt{x}} + C$$

$u = -4\sqrt{x}$

$u = -4x^{1/2}$

$du = -2x^{-1/2} dx$

$du = -2 \cdot \frac{1}{\sqrt{x}} dx$

$-\frac{1}{2} du = \frac{1}{\sqrt{x}} dx$

$$\int_1^\infty \frac{8e^{-4\sqrt{x}}}{7\sqrt{x}}\,dx = \lim_{b\to\infty} \int_1^b \frac{8e^{-4\sqrt{x}}}{7\sqrt{x}}\,dx$$

$$= \lim_{b\to\infty} -\frac{4}{7} e^{-4\sqrt{x}} \Big|_1^b$$

$$= \lim_{b\to\infty} -\frac{4}{7} \underbrace{e^{-4\sqrt{b}}} + \frac{4}{7} e^{-4} = -\frac{4}{7}(0) + \frac{4}{7} e^{-4}$$

$b \to \infty$

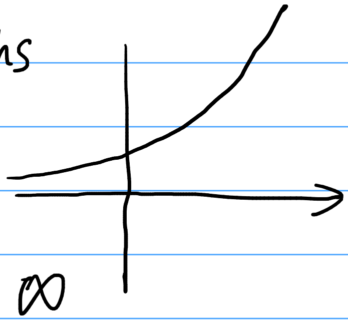
$\sqrt{b} \to \infty$

$-4\sqrt{b} \to -\infty$

$e^{-4\sqrt{b}}$ form $e^{-\infty}$ answer 0

Basic Graphs

$y = e^x$



$$\lim_{x\to\infty} e^x = \infty$$

$$\lim_{x\to-\infty} e^x = 0$$